First Year B.Sc (MRT) Degree Regular/Supplementary Examinations March 2025

Mathematics

Time: 3 Hours   **Total Marks: 100**

- *Answer all questions to the point neatly and legibly • Do not leave any blank pages between answers • Indicate the question number correctly for the answer in the margin space*
- *Answer all parts of a single question together •Leave sufficient space between answers*
- *Draw table/diagrams/flow charts wherever necessary.*
- *The use of a simple calculator is allowed, but using scientific calculator or sharing any calculator is strictly prohibited under any circumstance.*

Essay   **(2x20=40)**

1. a) The mean of the following distribution is 2.6. Find the value of k.

Values x	0	1	2	3	4	5
Frequency f	3	3	k	7	$k-1$	4

   b) Solve the system of equations
$$2x + y - z = 3$$
$$x + y + z = 1$$
$$x - 2y - 3z = 4$$
   using Cramer's Rule.

2. a) Using Simpson's rule evaluate
$$\int_0^2 \sqrt{1 + e^x}\, dx$$

   b) Evaluate the integral
$$\int \frac{\sec^2 \sqrt{x}\ \tan^2 \sqrt{x}}{\sqrt{x}}\, dx$$

   c) Express $(1 + i)^{18}$ in the form $a + ib$.

Short notes: **(8x5=40)**

3. Find $curl(\vec{F})$, where $\vec{F} = x^2 z\, \hat{\imath} + (e^y + xz)\, \hat{\jmath} + xyz\, \hat{k}$. What is $div(curl(\vec{F}\,))$.
4. If $r^2 = x^2 + y^2 + z^2$, find $grad\left(\frac{1}{r}\right)$.
5. Find the value of $tan\left(\frac{\pi}{8}\right)$.
6. Show that
$$\cos 2x \cos\left(\frac{x}{2}\right) - \cos 3x \cos\left(\frac{9x}{2}\right) = \sin 5x\, \sin\left(\frac{5x}{2}\right).$$
7. Find all the cube roots of i.
8. Consider a computer system with Poisson job arrival stream at an average of 2 jobs per minute. Determine the probability that in any one minute interval there will be at least 3 jobs.
9. Find the number of numbers greater than 10 those can be made using the digits 1,2,3,4 and 5.
10. Find the solution $y(x)$ of the differential equation
$$xy\frac{dy}{dx} = (x + 2)(y + 2)$$
   such that $y(1) = -1$.

(P.T.O)

Answer briefly: (10x2=20)

11. Find the value of n if

$$3^{-7} \times 3^{2n+3} = 3 \times 9^5 \div 3^5.$$

12. A wheel makes 360 <u>revolutions</u> in one minute. Through how many radians does it turn in one second?

13. Find

$$\lim_{x \to 9} \frac{9-x}{3 - \sqrt{x}}.$$

14. Find $\hat{\imath} \times (\hat{\jmath} \times \hat{k}) + \hat{\jmath} \times (\hat{k} \times \hat{\imath})$.

15. Find the angle between two vectors $3\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $2\hat{\imath} - \hat{\jmath} + \hat{k}$.

16. Find the modulus and amplitude of the complex number

$$\frac{(1 + 2i)^2}{1 - i}.$$

17. Three dice are rolled together. What is the probability of getting at least one '6'?

18. Prove the linearity property of Fourier Transform.

19. Find the Laplace transform of $f(t) = e^{2t} \sin 3t$.

20. Evaluate

$$\int_0^2 \sqrt{8x + 9}\, dx.$$

****